Преглед НЦД 12 (2008), 32–36

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OPTIMIZATION OF MUSIC DIGITIZATION

Abstract: The use of microtonal music scales (other than those, which is based on the equal tempered twelve semitones) is discussed. The western musical tuning notation is widely used now, but it is not quite adequate to record some modern genres (e.g. rock music) as well as many ancient and/or non-European compositions (E.g. Byzantine or Indian music). A mathematical approach based on solving simultaneous Diophantine approximation by the use of linear programming software is proposed and some experimental results are presented. An application towards the optimization of MIDI computer file standard is outlined.

Keywords: Digitization of music, Microtonal music, Simultaneous Diophantine approximation.

1. Introduction

The present-day process of wide-spread digitizing of the world cultural heritage puts some questions concerning the aspects of optimality for the formats used. The study object below is the transformation of musical notations into computer friendly forms. The musical notations are coded instructions for musicians to perform music. They represent events in a stream of time. Internationally, several written notations are in widespread use, such as Western staff notation, Chinese Jianpu notation, sol-fa, many instrument-specific notations such as guitar tablature and Japanese koto notation, and the Braille music for the blind, etc. [1]. An example of a piece of traditional notation is presented in Fig. 1, starting notes of the popular Bulgarian Hymn in Honor of Saints Cyril and Methodius.

Fig. 1. An example of traditional musical notation

Beside notations developed for human readers and performers, there are also many computer oriented representations of music designed to either be turned into conventional notation, or read directly by the computer.

There are a great many software programs designed to produce musical notation. Also, there are many file formats used to store musical information that the software can convert into notation, sound, or into some other usable form. In a sense, these file formats are a "notation" for computers.

The most common musical file format is probably the MIDI file format, which stores pitch and timing information about music (as well as velocity, volume, pitch bend, and modulation) and can be used to control a MIDI instrument which will produce the specified sound [2].

2. Microtonal music

Most of the music we hear is based on a system called "12 tone equal temperament" (12TET), where the octave is divided into 12 equal parts. Microtonal music is generally defined as any music that is not 12TET. Some folks base their music on the harmonic series, some divide the octave into 19 or 31 equal parts, some divide the octave into 43 unequal parts, etc.

The 12TET is the standard tuning for the Western music since the mid 19th-century, but there are a variety of artistic, theoretical, historical and philosophical reasons through which musicians may be drawn to "other intervals". As a result there are several disciplines, each of them studying "microtonality". Much non-Western music is microtonal: classical music from India, Turkey, Arabia and Persia, gamelan music from Indonesia, xylophone music from Africa, Byzantine liturgical music, folk music from the Caucasus, Middle- and Eastern Europe, "Trio Bulgarka – The Mystery Of Bulgarian Voices", etc.

3. The MIDI Tuning Standard

If f is a frequency in Hertz, then we may compute a corresponding logarithmic frequency value by

$$d = 69 + 12\log_2(f/440)$$

$$f = 440 \times 2^{(d-69)/12}$$

The pitch values d are presented in a form of three-digit integers of base 128 (each digit in hexadecimal notation by 00 through 7F, i.e., from 0 to 127 in base 10). The first byte digit represents the MIDI note, or integer notation, value. The next two digits allow the semitone to be divided into $128^2 = 2^{14} = 16384$ parts, which means the octave is divided into 196608 equal parts. This is far below the threshold of human pitch perception and therefore allows a very accurate representation of pitch. But this also contains some excess of useless information, that we want to reduce.

4. Toward optimization of MIDI-like computer files

4.1. Simultaneous Diophantine approximation. Given n positive real numbers, r_1, r_2, …, r_n, find non-negative integers, p_1, p_2, …, p_n, and q, such that the value of $|p_i/q - r_i|$ is "possibly small" for all $i = 1, 2, \ldots, n$, and q is "not so great":

$$\min \max_i \left| \frac{p_i}{q} - r_i \right| .$$

In the last years, there are published methods to solve the above problem, after the appropriate mathematical assumptions [3, 4, 5].

4.2. Linear Programming. Well known and widely used in many applications, the method of solving problems of (integer) linear programming is well equipped now with a sophisticated software. For example, we may use the freely distributed program lp_solve, which is originated at the Technical University in Eindhoven, The Netherlands, by Michel

Berkelaar. Under GNU Lesser General Public License that software is maintained now in the group http://groups.yahoo.com/group/lp_solve/.

4.3. Converting. The solving of a problem of Simultaneous Diophantine approximation can be transformed into the integer linear programming task:

Minimize w under conditions:

$0 < p_i < q,$
$-w \leq q\, r_i - p_i \leq w,$
$w > 0,$

where

$q > 0$ is fixed,
$r_i > 0$ are given ($i = 1, 2, \ldots, n$), and
p_i are integers for $i = 1, 2, \ldots, n$.

4.4. Pythagorean tuning. The classical problem leading to the contemporary twelve semitones equal tempered scale is how to approximate Pythagorean fractions: 2/1, 3/2, 4/3, 5/4, 6/5, 7/6, 8/7, 9/8, …, or fractions in the form $(i + 1)/i$, $i = 1, 2, \ldots$. These fractions are supposed to be “pleasantly sounded”; when two physical sounds are reproduced with frequencies that related each other as a Pythagorean fraction, we hear them in harmony. The frequency of a sound that is produced by a vibrating string is determined by the string length (Fig. 2).

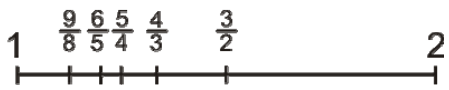

Fig. 2. Pythagorean fractions

We may try to find an approximation of the form:

$$\frac{i+1}{i} \approx 2^{\frac{p_i}{q}},$$

or

$$\log_2 \frac{i+1}{i} \approx \frac{p_i}{q}$$

Let us denote

$$r_i = \log_2 \frac{i+1}{i}, \quad i = 1, 2, \ldots, 8$$

and solve the corresponding linear programming problem, probing for different values of q.

4.5. Numerical experiments. For values $q = 12$ and $q = 19$ the following results are obtained (Table 1):

i	$i+1$	$(i+1)/i$	P	q	$2^{p/q}$	p	q	$2^{p/q}$
1	2	2.0000	12	12	2.0000	19	19	2.0000
2	3	1.5000	7	12	1.4983	11	19	1.4938
3	4	1.3333	5	12	1.3348	8	19	1.3389
4	5	1.2500	4	12	1.2599	6	19	1.2447
5	6	1.2000	3	12	1.1892	5	19	1.2001
6	7	*1.1667*	*3*	*12*	*1.1892*	*4*	*19*	*1.1571*
7	8	*1.1429*	*2*	*12*	*1.1225*	*4*	*19*	*1.1571*
8	9	1.1250	2	12	1.1225	3	19	1.1157

Table 1. Numerical results

The case $q = 12$ corresponds to the twelve semitones equal tempered scale and the values of computed approximations are shown on Fig. 3, where the decimal fractions are approximate values of $2^{2/12}$, $2^{3/12}$, $2^{4/12}$, $2^{5/12}$ and $2^{7/12}$, respectively.

The approximation error for different values of q is given on Fig. 4.

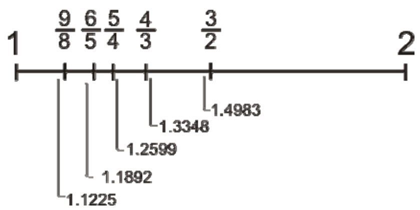

Fig. 3. Pythagorean fractions and their approximations with $q = 12$.

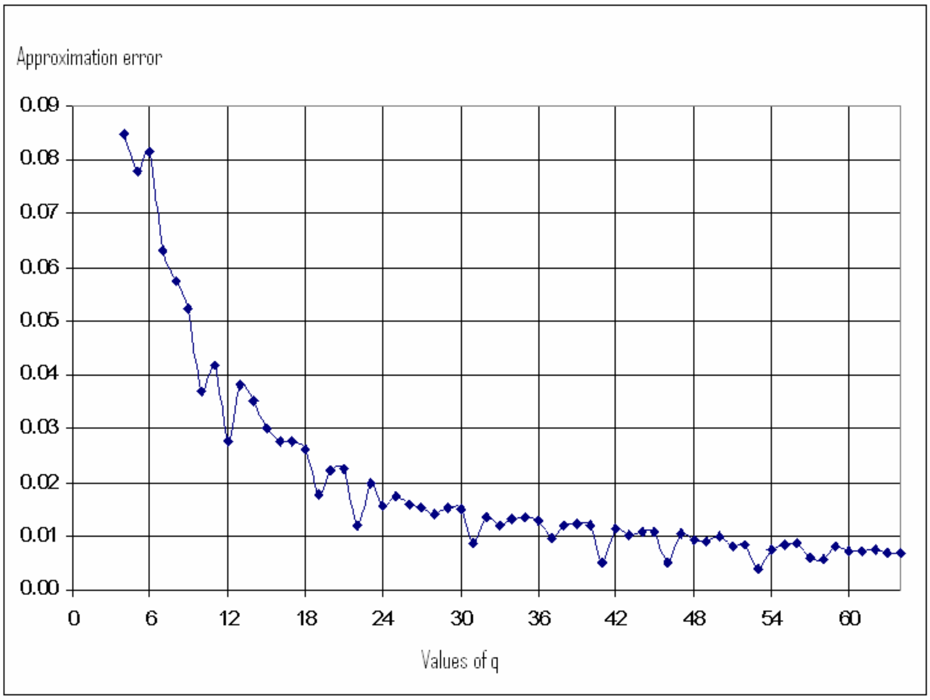

Fig. 4. The approximation error for different values of q

5. Conclusions. The proposed approach of finding approximations may serve to improve frequency coding in some digitizing standard (e.g. MIDI) for musical pieces. The contemporary equal tempered musical notation is a standard, but it is not enough adequate to record some modern genres written in microtonal notation (e.g. rock music) as well as many ancient and/or non-European compositions (E.g. Byzantine or Indian music). For an example, the Byzantine music is based on 72-tone equal temperament [6] and the above described approach may serve to transform with a minimal deflection to the 12-tone equal temperament scale or to find some intermediate scale.

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